SECTION 3.3
EXERCISE #10

3.2.10 Find the coordinate vector of $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ in M_2 (the vector space of all 2×2 matrices) relative to the ordered basis

$$B = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right).$$

<u>Solution</u>

Well, we coefficients $r_1, r_2, r_3, r_4 \in \mathbb{R}$ such that

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} = r_1 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + r_2 \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} + r_3 \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} + r_4 \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} r_3 & r_1 - r_2 - r_3 + r_4 \\ r_1 & 3r_3 + r_4 \end{bmatrix}.$$ So we need

$$\begin{aligned} r_3 &= 1 \\ r_1 &= 3 \\ r_1 - r_2 - r_3 + r_4 &= -2 \\ 3r_3 + r_4 &= 4. \end{aligned}$$

Notice we "clearly" have $r_1 = 3$ and $r_3 = 1$. So from the last equation $3(1) + r_4 = 4$ and $r_4 = 1$.

From the 3rd equation

$r_1 - r_2 - r_3 + r_4 = (3) - r_2 - (1) + (1) = -2$ and so $r_2 = 5$. Hence the desired coordinate vector is $\boxed{A_B = [r_1, r_2, r_3, r_4] = [3, 5, 1, 1].}$ □