SECTION 3.3
NUMBER 5

3.3.5 Find the coordinate vector of $\vec{v} = [3, 13, -1]$ relative to the ordered basis
$B = ([1, 3, -2], [4, 1, 3], [-1, 2, 0])$.

<u>Solution</u>

Well, we need to express $\vec{v}$ as a linear combination of the given basis elements. We'll have the coordinate vector
$\vec{v}_B = [v_1, v_2, v_3]$ where
$\vec{v} = v_1 \vec{b}_1 + v_2 \vec{b}_2 + v_3 \vec{b}_3$.

With A as a matrix whose columns are $\vec{b}_1, \vec{b}_2, \vec{b}_3$ we need to solve $A\vec{v}_B = \vec{v}$:

$$[A|\vec{v}] = \left[\begin{array}{ccc|c} 1 & 4 & -1 & 3 \\ 3 & 1 & 2 & 13 \\ -2 & 3 & 0 & -1 \end{array}\right] \xrightarrow[R_3 \to R_3 + 2R_1]{R_2 \to R_2 - 3R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 3 \\ 0 & -11 & 5 & 4 \\ 0 & 11 & -2 & 5 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 4 & -1 & 3 \\ 0 & -11 & 5 & 4 \\ 0 & 0 & 3 & 9 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3/(3)} \left[\begin{array}{ccc|c} 1 & 4 & -1 & 3 \\ 0 & -11 & 5 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2 \to R_2 - 5R_3]{R_1 \to R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 4 & 0 & 6 \\ 0 & -11 & 0 & -11 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2/(-11)} \left[\begin{array}{ccc|c} 1 & 4 & 0 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1 \to R_1 - 4R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

So $\boxed{\vec{v}_B = [2, 1, 3],}$ □