SECTION 3.3
NUMBER 9

3.3.9 Find the coordinate vector of $p(x) = x + x^4 \in P_4$ relative to the ordered basis
$B' = (1,\ 2x-1,\ x^3+x^4,\ 2x^3,\ x^2+2)$.

Solution

Introduce ordered basis $B = (x^4, x^3, x^2, x, 1)$. The coordinate vector relative to B of the coordinate vectors in B' are

$1_B = [0,0,0,0,1]$

$(2x-1)_B = [0,0,0,2,-1]$

$(x^3+x^4)_B = [1,1,0,0,0]$

$(2x^3)_B = [0,2,0,0,0]$

$(x^2+2)_B = [0,0,1,0,2]$.

Also, $p(x)_B = (x+x^4)_B = [1,0,0,1,0]$.
To find the coefficients of the basis vectors which produce $p(x)$, we see if $p(x)_B$ is in the column of matrix A with columns given by the coordinate vectors relative to B of the vectors in B':

$$\left[\begin{array}{ccccc|c} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 2 & 0 \end{array}\right] \underset{\sim}{\text{WA}} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1/2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

So, the coordinate $\boxed{p(x)_{B'} = [1/2, 1/2, 1, -1/2, 0].}$ □

CONT. →

SECTION 3.3
EXAMPLE

Find a basis for
$sp(1, 4x+3, 3x-4, x^2+2, x-x^2)$ in P_2.
Use ordered bases.

Solution

Introduce the ordered bases for P_2 of $B = (x^2, x, 1)$. Then the given vectors have coordinate vectors relative to order basis B of

$(1)_B = [0, 0, 1]$
$(4x+3)_B = [0, 4, 3]$
$(3x-4)_B = [0, 3, -4]$
$(x^2+2)_B = [1, 0, 2]$
$(x-x^2)_B = [-1, 1, 0]$

Well, lets make a matrix A with these vectors as its columns. Then the span of the columns of A are the column space of A; we find a basis for the column space of a matrix by row reducing A to H and considering pivots (see Note 2.2.A(2)).

So

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 \\ 0 & 4 & 3 & 0 & 1 \\ 1 & 3 & -4 & 2 & 0 \end{bmatrix} \overset{\text{RA}}{\sim} \begin{bmatrix} \boxed{1} & 0 & \frac{-25}{4} & 0 & \frac{5}{4} \\ 0 & \boxed{1} & \frac{3}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & \boxed{1} & -1 \end{bmatrix} = H.$$

So, a basis for the column space of A is given by the first, second, and fourth columns of A; ergo, a basis for the given span is given by the first, second and fourth vector:

$\boxed{\{1, 4x+3, x^2+2\}}$ □