SECTION 3.4

NUMBER 39

3.4.39 Let $\vec{v}$ and $\vec{w}$ be independent vectors in V, and let $T: V \to V'$ be a one-to-one linear transformation of V into V'. Prove that $T(\vec{v})$ and $T(\vec{w})$ are independent vectors in V'.

<u>Proof</u>

Let r and s be scalars such that $rT(\vec{v}) + sT(\vec{w}) = \vec{0}$. Since T is linear then by Definition 3.9,

$rT(\vec{v}) + sT(\vec{w}) = T(r\vec{v}) + T(s\vec{w})$, by "Preservation of Scalar Multiplication"

$= T(r\vec{v} + s\vec{w})$ by "Preservation of Addition".

Now by Theorem 3.5 (1), "Preservation of zero," we know that $T(\vec{0}) = \vec{0}'$ where $\vec{0}'$ is the zero vector in V'. Since T is one-to-one then by Corollary 3.4.A, "One-to-One Kernel," $\ker(T) = \vec{0}$ and so the only element of V mapped to $\vec{0}'$ by T is $\vec{0}$. Since $T(r\vec{v} + s\vec{w}) = rT(\vec{v}) + sT(\vec{w}) = \vec{0}$, then it must be that $r\vec{v} + s\vec{w} = \vec{0}$. Since $\vec{v}$ and $\vec{w}$ are independent then this implies by Definition 3.5, "Linear Dependence and Independence", that $r = s = 0$. Therefore, again by Definition 3.5, $T(\vec{v})$ and $T(\vec{W})$ are independent. ∎