SECTION 3.4
NUMBER 45

3.4.45 Let V and V' be finite dimensional vector spaces. Let $L(V,V')$ denote the set of all linear transformations mapping V to V'. Then $L(V,V')$ is a vector space (see "Summary Item 5" in Section 3.1). Let B and B' be ordered bases of V and V', respectively. Describe the matrix representation of T_1+T_2 and rT_1, where $r \in \mathbb{R}$, in terms of the matrix representations of T_1 and T_2 relative to B, B'.

<u>Solution</u>

Let A_1 be the matrix representation of T_1 relative to B, B' and let A_2 be the matrix representation of T_2 relative to B, B' (see Definition 3.11). So for any $\vec{v} \in V$ we have $T_1(\vec{v})_{B'} = A_1 \vec{v}_B$ and $T_2(\vec{v})_{B'} = A_2 \vec{v}_B$. Then $(T_1+T_2)\vec{v} = T_1(\vec{v}) + T_2(\vec{v})$ by the definition of addition in $L(V,V')$ (see Exercise 43) and $((T_1+T_2)\vec{v})_{B'} = (T_1(\vec{v})+T_2(\vec{v}))_{B'}$

$$= T_1(\vec{v})_{B'} + T_2(\vec{v})_{B'} = A_1\vec{v}_B + A_2\vec{v}_B$$

$$= (A_1+A_2)\vec{v}_B.$$

$\boxed{\text{Therefore the matrix representation of } T_1+T_2 \text{ is } A_1+A_2.}$

Next, for any $\vec{v} \in V$ we have $((rT_1)(\vec{v}))_{B'} = (r(T_1(\vec{v})))_{B'}$ by the definition of scalar multiplication (see Exercise 44)

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and $(r\,T_1(\vec{v}))_{B'} = r\,(T_1(\vec{v}))_{B'} = r(A_1\,\vec{v}_B) = (rA_1)\,\vec{v}_B$.

$\boxed{\text{So the matrix representation of } rT_1 \text{ is } rA_1.}$ $\square$