SECTION 3.4
NUMBER 5

3.4.5 Consider $T: F \to F$ defined as $T(f) = -f$. Is $T$ linear?

Solution

Well, by Definition 3.9, we need to check ① $T(f+g) = T(f) + T(g)$, ② $T(rf) = rT(f)$ for all $f, g \in F$ and for all $r \in \mathbb{R}$.

First, let $f, g \in F$. Then

$$T(f+g) = -(f+g) = -f - g = (-f) + (-g) = T(f) + T(g).$$

Second, let $r \in \mathbb{R}$. Then

$$T(rf) = -(rf) = r(-f) = rT(f).$$

So YES, $T$ is linear. □