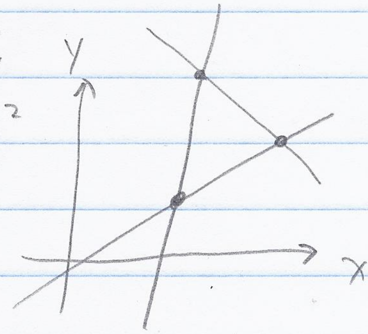
SECTION 4.1
NUMBER 29

4.1.29 Find the area in $\mathbb{R}^2$ of the triangle bounded by $y = x$, $y = -3x + 8$, and $3y + 5x = 0$.

<u>Solution</u>

Schematically, in $\mathbb{R}^2$ we have:



So lets find the of intersection!

Now $y = x$ and $y = -3x + 8$ intersect when $x = -3x + 8$ or $x = 2$ (and $y = 2$).

Next, $y = x$ and $3y + 5x = 0$ intersect when $3(x) + 5x = 0$ or $x = 0$ (and $y = 0$).

Thirdly, $y = -3x + 8$ and $3y + 5x = 0$ intersect when $3(-3x + 8) + 5x = 0$ or $-4x + 24 = 0$ or $x = 6$ (and $y = -10$).

So the lines intersect at $(2, 2)$, $(0, 0)$ and $(6, -10)$. These three points determine the two vectors (say) $\vec{a} = [2, 2]$ and $\vec{b} = [6, -10]$.

So the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$ is $|\det(A)|$ where $A = \begin{bmatrix} 2 & 2 \\ 6 & -10 \end{bmatrix}$

$$|\det(A)| = \left| \begin{vmatrix} 2 & 2 \\ 6 & -10 \end{vmatrix} \right| = |(2)(-10) - (2)(6)|$$

$$= |-32| = 32.$$

So the area of the triangle is $\frac{32}{2} = \boxed{16.}$ $\square$