SECTION 4.2
NUMBER 19

4.2.19 If A is a 3×3 matrix with $\det(A) = 2$ then $\det(A^{-1}) = ?$

<u>Solution</u>

Recall that the "Multiplicative Property" (Theorem 4.2) implies that

$$\det(AB) = \det(A)\det(B).$$

So, <u>if</u> A^{-1} exists, then

$$\det(I) = \det(AA^{-1}) = \det(A)\det(A^{-1}). \quad (*)$$

We know that $\det(I) = 1$ (expand along the first row to get

$$\underbrace{\begin{vmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{vmatrix}}_{n \times n} = +(1)\underbrace{\begin{vmatrix} 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ & & \ddots & \\ 0 & 0 & & 1 \end{vmatrix}}_{(n-1)\times(n-1)} - 0 + 0 - 0 + \cdots 0$$

$$= (1)(1)\underbrace{\begin{vmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \cdots & 1 \end{vmatrix}}_{(n-2)\times(n-2)} = \cdots = (1)(1)\cdots(1) = 1 \Big).$$

So by $(*)$,
$$1 = \det(A)\det(A^{-1})$$
$$= 2\det(A^{-1})$$
since $\det(A) = 2$

and so $\boxed{\det(A^{-1}) = 1/2}$ $\square$

<u>Note</u> In Exercise 4.2.31 we show more generally, $\det(A^{-1}) = \dfrac{1}{\det(A)}$.