SECTION 4.2
NUMBER 29

4.2.29 Find λ such that $\begin{bmatrix} 1-\lambda & 0 & 2 \\ 0 & 4-\lambda & 3 \\ 0 & 4 & -\lambda \end{bmatrix}$ is singular.

<u>Solution</u>

Well, by Theorem 4.3, A is singular (that is, NOT invertible) if and only if $\det(A) = 0$. So we want

$$\begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 4-\lambda & 3 \\ 0 & 4 & -\lambda \end{vmatrix} = 0.$$ Well,

$$\begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 4-\lambda & 3 \\ 0 & 4 & -\lambda \end{vmatrix} = +(1-\lambda)\begin{vmatrix} 4-\lambda & 3 \\ 4 & -\lambda \end{vmatrix} - (0) + (0)$$

expanding along the first column

$$= (1-\lambda)\big((4-\lambda)(-\lambda) - (3)(4)\big)$$

$$= (1-\lambda)(-4\lambda + \lambda^2 - 12) = (1-\lambda)(\lambda-6)(\lambda+2),$$

so we need $(1-\lambda)(\lambda-6)(\lambda+2) = 0$,

or $\boxed{\lambda = 1 \text{ or } \lambda = 6 \text{ or } \lambda = -2.}$ $\square$