

SECTION 4.2
NUMBER 3

4.2.3

Find the determinant using cofactors of

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 1 & 2 \\ 1 & 4 & 1 \end{bmatrix}.$$

Solution

Let's find $\det(A)$ by expanding along the first column:

$$\det(A) = a_{11} a'_{11} + a_{21} a'_{21} + a_{31} a'_{31}.$$

Next,

$$a'_{11} = (-1)^{1+1} |A_{11}| = (-1)^2 \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} = +((1)(1) - (2)(4)) = -7$$

$$a'_{21} = (-1)^{2+1} |A_{21}| = (-1)^3 \begin{vmatrix} 2 & 4 \\ 4 & 1 \end{vmatrix} = -(-14) = 14$$

$$a'_{31} = (-1)^{3+1} |A_{31}| = (-1)^4 \begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = +(0) = 0.$$

So,

$$\det(A) = (3)(-7) + (0)(14) + (1)(0) = \boxed{-21}.$$