SECTION 4.3

NUMBER 3

4.3.3 Row reduce $A$ to an upper triangular matrix $H$ and then find $\det(A)$ for

$$A = \begin{bmatrix} 5 & 2 & 4 & 0 \\ 2 & -3 & -1 & 2 \\ 3 & -4 & 3 & 7 \\ 1 & -1 & 0 & 1 \end{bmatrix}.$$

<u>Solution</u> Well,

$$A \xrightarrow{R_1 \leftrightarrow R_4} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 2 & -3 & -1 & 2 \\ 3 & -4 & 3 & 7 \\ 5 & 2 & 4 & 0 \end{bmatrix} \xrightarrow[\substack{R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 5R_1}]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & -1 & 3 & 4 \\ 0 & 7 & 4 & -5 \end{bmatrix}$$

$$\xrightarrow[R_4 \to R_4 + 7R_2]{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & -3 & -5 \end{bmatrix} \xrightarrow{R_4 \to R_4 + \frac{3}{4}R_3} \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$

So

$$\boxed{H = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & -2 \end{bmatrix}.}$$

Next $A \sim H$ through a sequence of 6 row additions (which don't affect determinants) and $r = 1$ row interchange. So

$$\det(A) = (-1)^r \det(H) = (-1)^1\big((1)(-1)(4)(-2)\big)$$

$$= \boxed{-8.} \quad \square$$