SECTION 4.3
NUMBER 4

4.3.4 Find the determinant of $\begin{bmatrix} 3 & -5 & -1 & 7 \\ 0 & 3 & 1 & -6 \\ 2 & -5 & -1 & 8 \\ -8 & 8 & 2 & -9 \end{bmatrix} = A.$

<u>Solution</u>

Well, lets use row addition (which does not affect determinants):

$$A \xrightarrow[\substack{R_3 \to R_3 + R_2 \\ R_4 \to R_4 - 2R_2}]{R_1 \to R_1 + R_2} \begin{bmatrix} 3 & -2 & 0 & 1 \\ 0 & 3 & 1 & -6 \\ 2 & -2 & 0 & 2 \\ -8 & 2 & 0 & 3 \end{bmatrix}$$

$$\xrightarrow[R_4 \to R_4 - 3R_1]{R_3 \to R_3 - 2R_1} \begin{bmatrix} 3 & -2 & 0 & 1 \\ 0 & 3 & 1 & -6 \\ -4 & 2 & 0 & 0 \\ -17 & 8 & 0 & 0 \end{bmatrix} = H.$$

Since $A \sim H$ through a sequence of row addition, then $\det(A) = \det(H)$ by Theorem 4.2.A (5), so

$$\det(A) = \det(H) = \begin{vmatrix} 3 & -2 & 0 & 1 \\ 0 & 3 & 1 & -6 \\ -4 & 2 & 0 & 0 \\ -17 & 8 & 0 & 0 \end{vmatrix}$$

$$= +(0) - (1)\begin{vmatrix} 3 & -2 & 1 \\ -4 & 2 & 0 \\ -17 & 8 & 0 \end{vmatrix} + (0) - (0)$$

$$= -(1)\left((1)\begin{vmatrix} -4 & 2 \\ -17 & 8 \end{vmatrix} - (0) + (0) \right) = (-1)(1)\left((-4)(8) - (2)(-17)\right)$$

$$= -(-32 + 34) = \boxed{-2}. \quad \square$$