SECTION 5.2
EXERCISE 11

5.2.11 Determine whether this matrix is diagonalizable:

$$A = \begin{bmatrix} -1 & 4 & 2 & -7 \\ 0 & 5 & -3 & 6 \\ 0 & 0 & -5 & 1 \\ 0 & 0 & 0 & 11 \end{bmatrix}.$$

Solution

First, we find the eigenvalues of A by considering the characteristic polynomial $\det(A - \lambda I)$:

$$\det(A - \lambda I) = \begin{vmatrix} -1-\lambda & 4 & 2 & -7 \\ 0 & 5-\lambda & -3 & 6 \\ 0 & 0 & -5-\lambda & 1 \\ 0 & 0 & 0 & 11-\lambda \end{vmatrix}$$

$= (-1-\lambda)(5-\lambda)(-5-\lambda)(11-\lambda)$ since $A - \lambda I$ is upper triangular (see Page 255 Example 4).

Set $p(\lambda) = (-1-\lambda)(5-\lambda)(-5-\lambda)(11-\lambda) = 0$ and we have that the eigenvalues of A are $\lambda_1 = -5$, $\lambda_2 = -1$, $\lambda_3 = 5$, $\lambda_4 = 11$. Since 4×4 matrix A has 4 distinct eigenvalues, then by Theorem 5.3, "Independence of Eigenvectors", $\boxed{\text{matrix } A \text{ is diagonalizable.}}$ □