SECTION 5.2
EXERCISE #17

5.2.17 Prove that, for every square matrix A all of whose eigenvalues are real, the product of its eigenvalues is $\det(A)$.

<u>Proof</u>

Let the real eigenvalues of A be $\lambda_1, \lambda_2, \ldots, \lambda_n$ (not necessarily distinct). Since these are the roots of the characteristic polynomial then $p(\lambda) = \det(A - \lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda) \cdots (\lambda_n - \lambda)$ (remember that finding a root r of a polynomial produces a factor $(x - r)$ of the polynomial).
Now $p(\lambda)$ is just a polynomial in variable λ. If we set $\lambda = 0$ we get

$$p(0) = \det(A - 0I) = \det(A) = (\lambda_1 - 0)(\lambda_2 - 0) \cdots (\lambda_n - 0)$$
$$= \lambda_1 \lambda_2 \cdots \lambda_n.$$

So the product of the eigenvalues of A is $\det(A)$, as claimed. ∎

<u>Note</u> The result holds even if the eigenvalues of A are complex (as opposed to real). In fact, it even holds for matrix A with complex entries.