

SECTION 6.1
EXERCISE 11

6.1.11

Find the orthogonal complement W^\perp of $W = \text{sp}([2, 1, 3, 4], [1, 0, -2, 1])$ in \mathbb{R}^4 .

Solution

By Note 6.1.A, we create matrix A with the given vectors as ROW vectors and then find the nullspace of A by solving the system of equations $A\vec{x} = \vec{0}$.

So we have $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 1 & 0 & -2 & 1 \end{bmatrix}$ and consider

$$\left[\begin{array}{cccc|c} 2 & 1 & 3 & 4 & 0 \\ 1 & 0 & -2 & 1 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & 0 \\ 2 & 1 & 3 & 4 & 0 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 - 2R_1}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 7 & 2 & 0 \end{array} \right] \quad \text{or} \quad \begin{array}{l} x_1 - 2x_3 + x_4 = 0 \\ x_2 + 7x_3 + 2x_4 = 0 \end{array} \quad \text{or}$$

$$x_1 = 2x_3 - x_4 \quad \text{or with } r = x_3 \quad x_1 = 2r - s$$

$$x_2 = -7x_3 - 2x_4 \quad \text{and } s = x_4 \quad \text{or} \quad x_2 = -7r - 2s$$

$$x_3 = x_3 \quad \text{free variable} \quad x_3 = r$$

$$x_4 = x_4 \quad x_4 = s$$

$$\text{or} \quad \vec{x} = r \begin{bmatrix} 2 \\ -7 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix} \quad \text{where } r, s \in \mathbb{R}$$

So the nullspace of A and W^\perp is

$$\boxed{\text{sp}([2, -7, 1, 0], [-1, -2, 0, 1])} \quad \square$$