

SECTION 6.2

NUMBER 31

6.2.31 Let A be an $n \times n$ matrix. Prove that the column vectors of A are orthonormal if and only if the row vectors of A are orthonormal.

HINT: Use Exercise 30 which states: "Square matrix A has orthonormal column vectors if and only if A is invertible with inverse $A^{-1} = A^T$."

Proof

The column vectors of A are orthonormal if and only if A is invertible and $A^{-1} = A^T$ by Exercise 30. Now A is invertible with $A^{-1} = A^T$ if and only if $AA^T = I$. By Exercise 29 (with A and A^T interchanged) this holds if and only if the column vectors of A^T are orthonormal; that is, if and only if the row vectors of A are orthonormal. So the column vectors of A are orthonormal if and only if the row vectors of A are orthonormal. ■