

SECTION 6.3
NUMBER 29

6.3, 29 (a) Prove that a row interchange elementary matrix is orthogonal.

Proof

If E is an $n \times n$ row interchange elementary matrix, then E has the same rows as the $n \times n$ identity matrix (only in a different order).

Since the rows of I form an orthonormal basis for \mathbb{R}^n (in fact, they form the standard basis for \mathbb{R}^n),

then the rows of E form an orthonormal basis for \mathbb{R}^n .

So by Theorem 6.5, "Characterizing Properties of an Orthogonal Matrix" (the (1) implies (3) part), E is orthogonal. ■

(b) Let A be a matrix obtained by permuting (that is, changing the order of) the rows of the $n \times n$ identity matrix. Prove that A is an orthogonal matrix.

Proof

A has the same rows as the $n \times n$ identity matrix (only in a different order). Since the rows of I form an orthonormal basis for \mathbb{R}^n , then the rows of A form an orthonormal basis for \mathbb{R}^n . So by Theorem 6.5 (the (1) implies (3) part), A is orthogonal. ■