

SECTION 6.3

NUMBER 31

6.3.31 Let A and C be orthogonal $n \times n$ matrices.Prove that $C^{-1}AC$ is orthogonal.Proof

Since A and C are orthogonal then by Definition 6.4, "Orthogonal Matrix" (and the Note following it), $A^{-1} = A^T$ and $C^{-1} = C^T$.

Then

$$\begin{aligned}(C^{-1}AC)^{-1} &= C^{-1}A^{-1}(C^{-1})^{-1} \quad \text{by Theorem 1.20,} \\ &\quad \text{"Inverse Products"} \\ &= C^{-1}A^T C = C^T A^T C = (C^T A C)^T \\ &= (C^{-1}AC)^T.\end{aligned}$$

Therefore (again by Definition 6.4), $C^{-1}AC$ is orthogonal. ■