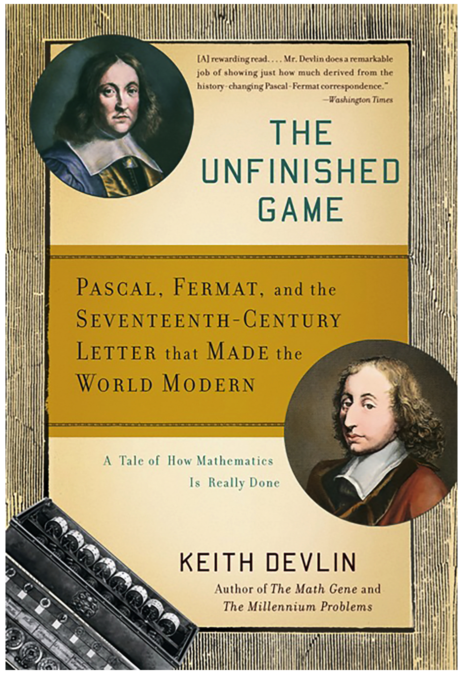
Chapter 2. Probability

Note. In this chapter, we introduce basic properties of probability and consider the probability of events over a finite sample space. In so doing, we introduce counting techniques related to finite sets. We define the conditional probability of one event *given* another. We introduce random variables (which are very fundamental objects in the study of probability and statistics) and consider some of their properties.

Note. Probability is a subtle art! It's history dates to the mid 17th century. In a letter from Blaise Pascal (June 19, 1623–August 19, 1662) to Pierre de Fermat (late 1607–December 6, 1607; both were French) dated August 24, 1654, a dice game which is interrupted before it can be finished is described and the question is posed as to how the pot should be split up without finishing the game. This lead to a discussion about how the game *might* have ended, were it allowed to be completed, and how *likely* the various endings were. At this point in history, there was still a philosophical (possibly even religious) resistance to making predictions about future events which were not predetermined. It was this correspondence that initiated a quantitative study of possible outcomes of games and, eventually, other events. This still took some time to formalize, and the terminology of modern probability is due to mathematician Jakob Bernoulli (January 6, 1655–August 16, 1705) in his *Ars Conjectandi* (The Art of Conjecturing) published after his death in 1713. The book is also viewed as a historical contribution to combinatorial theory. The story of Pascal and Fermat's correspondence is spelled out in Keith Devlin's popular-level book *The Unfinished Game — Pascal, Fermat, and the Seventeenth-*

Century Letter that Made the World Modern, New York: Basic Books (2008).



Note. In this course, we don't explore the various interpretations of the concept of "probability," but we do mention three such interpretations:

The Frequency Interpretation of Probability. The idea is that the probability of a specific outcome of an experiment is the relative frequency that the outcome occurs if the experiment is repeated a large number of times "under similar conditions."

The Classical Interpretation of Probability. This is based on the idea of equally likely outcomes of an experiment. Difficulties with this approach include the fact that "likely" is part of what we are trying to define, and this interpretation does not deal with non-likely outcomes.

The Subjective Interpretation of Probability. This is also called the personal interpretation. The probability of a possible outcome is based on the personal judgment of the likelihood of that outcome.

This definitions are from my online notes for Intermediate Probability and Statistics (not a formal ETSU class, but a sort-of "lite" version of Mathematical Statistics [STAT 4147/5147]) on Section 1.2. Interpretations of Probability.

Note. A class in modern probability theory goes well-beyond simple combinatorial methods of counting. Today, probability theory is a branch of analysis (well, probability is a branch of mathematics *itself*). To study modern probability theory requires a background in measure theory and integration (such as that found in ETSUs Real Analysis 1 and 2 [MATH 5210, MATH 5220]), and functional analysis (such as that found in ETSU's Fundamental's of Function Analysis [MATH 5740]). For your amusement, notes are online for this material: Real Analysis 1 (MATH 5210), Real Analysis 2 (MATH 5220), and Fundamentals of Functional Analysis (MATH 5740). Additional functional analysis material based on the real analysis text by Royden and Fitzpatrick is also posted online. I have notes in preparation for a Measure Theory Based Probability class (also not a formal ETSU class).

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