Chapter 11. Parametric Equations and Polar Coordinates

11.3. Polar Coordinates

Definition. We define the polar coordinates of a point $P(r, \theta)$ in the Cartesian plane by introducing an initial ray from the origin $O$ which lies along the $x$-axis. Point $P$ is then said to lie at $P(r, \theta)$ if either (1) it lies a distance $r$ ($r \geq 0$) along a ray which makes an angle of $\theta$ with the initial ray, or (2) it lies a distance $-r$ ($r \leq 0$) along a ray which makes an angle of $\theta$ with the initial ray. Coordinate $r$ gives the directed distance of $P$ from $O$.

Note. Due to the the fact that coterminal rays can be represented with different values of $\theta$, then a point in the Cartesian plane can have multiple representations in polar coordinates.

Example. Page 648, number 4c.
**Note.** If we hold $r$ fixed at a constant value, $r = a \neq 0$, then the point $P(r, \theta)$ will lie $|a|$ units from the origin $O$. As $\theta$ varies over any interval of length $2\pi$, $P$ then traces a circle of radius $|a|$ centered at $O$. If we hold $\theta$ fixed at a constant value $\theta = \theta_0$ and let $r$ vary between $-\infty$ and $\infty$, the point $P(r, \theta)$ traces the line through $O$ that makes an angle of measure $\theta_0$ with the initial ray. In general, we can relate Cartesian $(x, y)$ coordinates to polar coordinates $P(r, \theta)$ as:

$$x = r \cos \theta, \quad y = r \sin \theta, \quad r^2 = x^2 + y^2, \quad \tan \theta = \frac{y}{x}.$$

![Diagram](image)

Figure 11.24, page 647

**Examples.** Page 649, numbers 36 and 62.