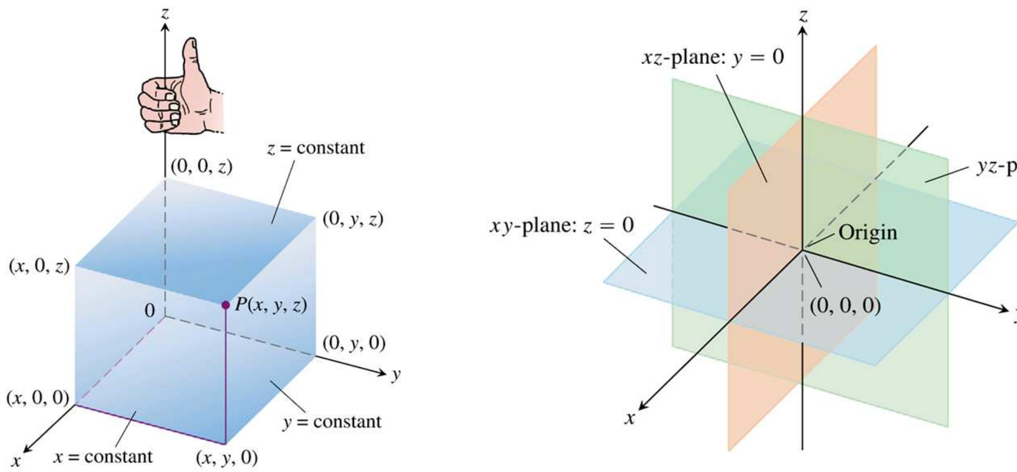
Chapter 12. Vectors and the Geometry of Space

12.1. Three-Dimensional Coordinate Systems

Note. You have never legitimately been into three dimensional space in your calculus career. You dealt with solids of revolution in Calculus 2, but this was approached in a restricted way that still based things on two-dimensional coordinate systems. In this section, we legitimately mathematically enter three dimensions (physically, you lived there your whole life)!

Definition. We introduce *three-dimensional Cartesian coordinates*, (x, y, z), by considering three mutually orthogonal (i.e., perpendicular) coordinate axes, the x-axis, the y-axis, and the z-axis. We do so in such a way as to determine a *right-hand* coordinate system. If you curl the fingers of your right hand from the positive x-axis to the positive y-axis, then your thumb will point in the direction of the positive z-axis. Such a system determines three coordinate planes, the xy-plane, the xz-plane, and the yz-plane. For point $P(x, y, z)$, coordinate x represents the distance of P from the yz-plane, coordinate y represents the distance of P from the xz-plane, and coordinate z represents the distance of P from the xy-plane. The coordinate planes divide three-dimensional space into eight *octants*, depending on the signs of the coordinates of the points in that octant.

The *first octant* contains all points with positive coordinates.



Figures 12.1 and 12.2, pages 678 and 679

Example. Page 681, number 6

Note. It follows from the Pythagorean Theorem that distance is measured in three-dimensional space between points $P_x(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$P_1P_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

It follows from this formula for distance that the formula for a sphere of radius a and center (x_0, y_0, z_0) is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = 1^2.$$

Examples. Page 681, number 24a; page 682, number 30a; page 682, number 64.