Chapter 15. Multiple Integrals

15.3. Area by Double Integration

Note. If we take $f(x, y) = 1$ in the definition of the double integral over a region $R$ in the preceding section, the Riemann sums reduce to

$$S_n = \sum_{k=1}^{n} f(x_k, y_k) \Delta A_k = \sum_{k=1}^{n} \Delta A_k.$$

This is simply the sum of the areas of the small rectangles in the partition of $R$, and approximates what we would like to call the area of $R$.

Definition. The area of a closed, bounded plane region $R$ is

$$A = \iint_{R} dA.$$

Examples. Page 870, numbers 8 and 14.

Definition. The average value of $f(x, y)$ over region $R$ is

$$\frac{1}{\text{area of } R} \iint_{R} f \, dA.$$

Example. Page 870, number 20.