Calculus 3 Test 1 — Spring 2011

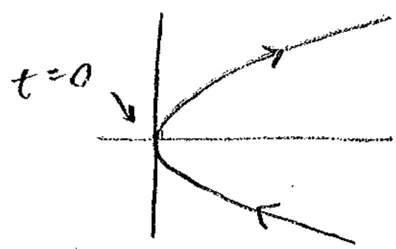
NAME KEY STUDENT NUMBER ____________________

SHOW ALL WORK!!! Partial credit will only be given for answers which are *partially correct*! Be clear and convince me that you understand what is going on. Communicate with me; write words! Use equal signs when things are equal (and don't use them when things are not equal). Be sure to include all necessary symbols, such as equal signs. If applicable, put your answer in the box provided. Each numbered problem is worth 12 points and the bonus problem is worth 12 points. **Put your calculators away!!! This is a *math* test!!!**

p634 #15

1. Consider the parametric equations $x = \sec^2(t) - 1$, $y = \tan t$, $t \in (-\pi/2, \pi/2)$. Identify the curve by finding a Cartesian equation for it. Graph the Cartesian equation.

Well, $\tan^2(\theta) = \sec^2\theta - 1$, so $y^2 = x$ and this is a parabola. The graph is



$\boxed{\text{parabola}}$

p643 #25

2. Find the length of the curve $x = \cos t$, $y = t + \sin t$, $t \in [0, \pi]$.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^\pi \sqrt{(-\sin(t))^2 + (1+\cos(t))^2}\,dt$$

$$= \int_0^\pi \sqrt{\sin^2(t) + 1 + 2\cos(t) + \cos^2(t)}\,dt = \int_0^\pi \sqrt{2(1+\cos(t))}\,dt$$

$$= \int_0^\pi \sqrt{2 \cdot 2\cos^2(t/2)}\,dt \quad \text{since } \cos^2(x) = \frac{1+\cos(2x)}{2}$$

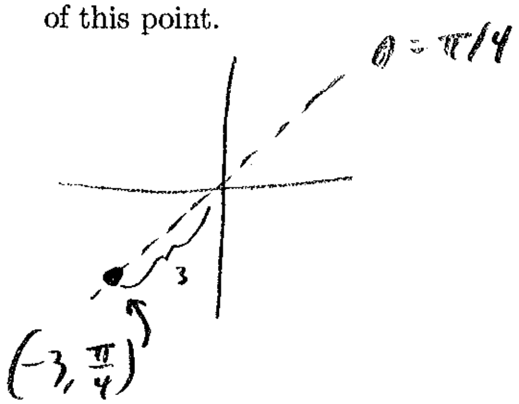
$$= \int_0^\pi 2\,|\cos(t/2)|\,dt = \int_0^\pi 2\cos(t/2)\,dt \quad \text{since } \cos(t/2) \geq 0 \text{ for } t \in [0,\pi]$$

$$= 4\sin\left(\tfrac{t}{2}\right)\Big|_0^\pi = 4\sin\left(\tfrac{\pi}{2}\right) - 4\sin(0) = 4$$

$\boxed{4}$

3. Plot the point in polar coordinates $(r, \theta) = (-3, \pi/4)$. Find all polar coordinate representations of this point.

p652 #17



The other representations are

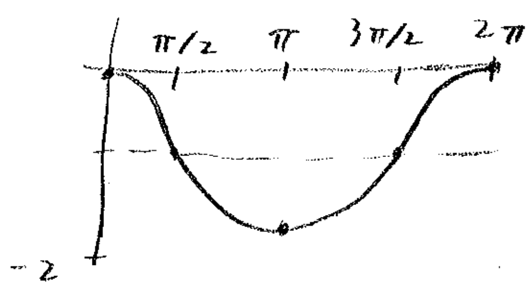
$\left(-3, \frac{\pi}{4} + 2n\pi\right)$ for $n \in \mathbb{Z}$ and

$\left(3, \frac{5\pi}{4} + 2n\pi\right)$ for $n \in \mathbb{Z}$.

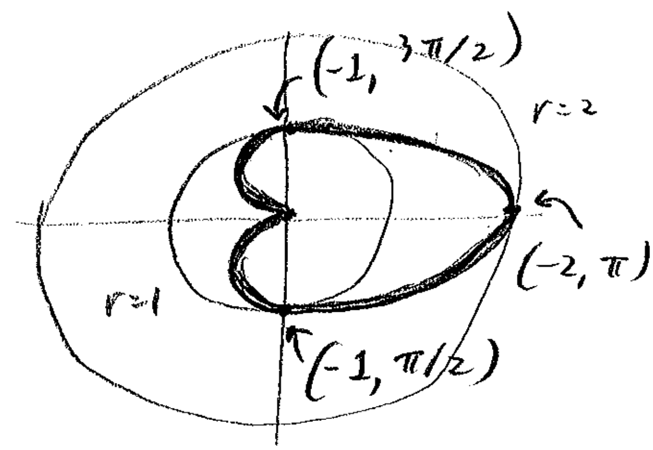
4. Sketch the curve $r = -1 + \cos\theta$. HINT: Graph $y = -1 + \cos x$ in rectangular coordinates and use this as a reference.

p648 #46

Well, we have



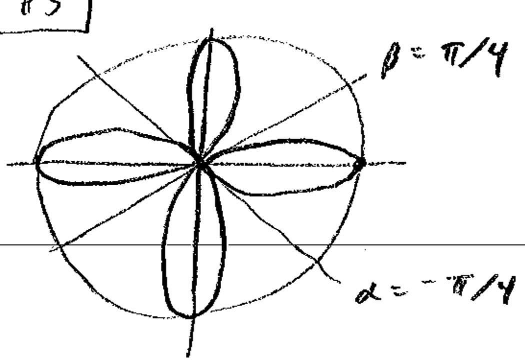
So:



5. Find the area in one leaf of the four-leaved rose $r = \cos(2\theta)$.

p656 #5

The graph is



So, $A = \int_{-\pi/4}^{\pi/4} \frac{1}{2}\left(\cos(2\theta)\right)^2 d\theta$

$$= \int_{-\pi/4}^{\pi/4} \frac{1}{2}\left(\frac{1+\cos(4\theta)}{2}\right) d\theta = \frac{1}{4}\left(\theta + \frac{1}{4}\sin(4\theta)\right)\Big|_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{4}\left(\frac{\pi}{4} + \frac{1}{4}\sin(\pi)\right) - \frac{1}{4}\left(-\frac{\pi}{4} + \frac{1}{4}\sin(-\pi)\right)$$

$$= \boxed{\frac{\pi}{8}}$$

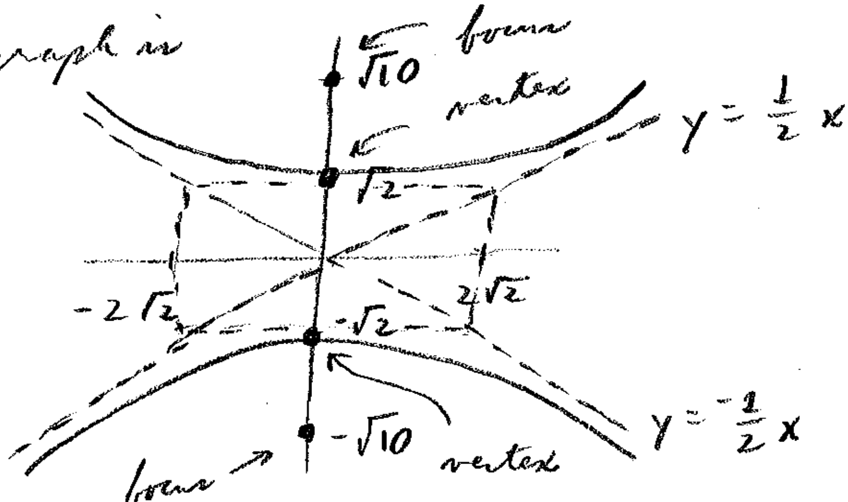
6. Consider the conic section $8y^2 - 2x^2 = 16$. Put the conic in standard form, graph the conic, any asymptotes, vertices, and foci.

p 664 #33

Well, $\frac{y^2}{2} - \frac{x^2}{8} = 1$. This is a hyperbola with

$a = \sqrt{2}$, $b = \sqrt{8} = 2\sqrt{2}$ and so $c^2 = a^2 + b^2 = 2 + 8 = 10$, so $c = \sqrt{10}$.

The graph is



$\boxed{\text{vertices } (0, \pm\sqrt{2}) \quad \text{foci } (0, \pm\sqrt{10})}$

7. Consider the ellipse centered at the origin with foci $(0, \pm 3)$ and eccentricity 0.5. Find the ellipse's standard-form equation in Cartesian coordinates.

p 671 #9

Well, foci are at $(0, \pm c)$, so $c = 3$. $e = \frac{c}{a} = \frac{1}{2}$,

so $a = 6$. Next, $c^2 = a^2 - b^2 \Rightarrow (3)^2 = (6)^2 - (b)^2 \Rightarrow b = \sqrt{27} = 3\sqrt{3}$.

So standard form is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ since foci are on y-axis

or $\boxed{\frac{x^2}{27} + \frac{y^2}{36} = 1}$

8. Show that the point $P(3, 1, 2)$ is equidistant from the points $A(2, -1, 3)$ and $B(4, 3, 1)$.

p 682 #62

Well, distance is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$,

so $PA = \sqrt{(3-2)^2 + (1-(-1))^2 + (2-3)^2} = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{6}$

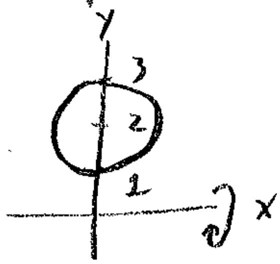
and $PB = \sqrt{(3-4)^2 + (1-3)^2 + (2-1)^2} = \sqrt{(-1)^2 + (-2)^2 + (-1)^2} = \sqrt{6}$,

so $PA = PB = \sqrt{6}$.

Bonus. Suppose the parametric curve $x = \cos t$, $y = 2 + \sin t$ for $t \in [0, 2\pi]$ is revolved about the x-axis. What is the resulting surface area?

$\boxed{\begin{array}{c} p643 \\ \#31 \end{array}}$

Well, $x^2+(y-2)^2=1$, so the graph is



and $S = \displaystyle\int_{t=a}^{t=b} 2\pi \text{ radius } (\text{arc length})$

$$= \int_{t=0}^{t=2\pi} 2\pi y \, ds = \int_{t=0}^{t=2\pi} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$$= \int_0^{2\pi} 2\pi (2+\sin t)\sqrt{(-\sin t)^2 + (\cos t)^2} \, dt = \int_0^{2\pi} 2\pi (2+\sin t) \, dt$$

$$= 2\pi (2t - \cos t)\Big|_0^{2\pi} = 2\pi(4\pi - \cos(2\pi)) - 2\pi(0 - \cos 0)$$

$$= \boxed{8\pi^2}$$