

# Calculus 3, Chapter 14 Study Guide

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The following is a *brief* list of topics covered in Chapter 14 of *Thomas' Calculus*. Test questions will be chosen directly from the text. This list is not meant to be comprehensive, but only gives a list of several important topics. I reserve the right to ask you definitions and theorems on the tests. If I do so, then I will choose from the **bold-faced** items below.

**14.1 Functions of Several Variables.** Function of several variables, domain, range, dependent variable, independent variables, **interior point/boundary point/limit point of a region in the  $xy$ -plane, open region/closed region/closure of a region in the  $xy$ -plane, bounded/unbounded region in the  $xy$ -plane**, graphing or describing domains, **level curve/level surface**.

**14.2 Limits and Continuity in Higher Dimensions.** Thomas' Definition of Limit (and its problems), Alternate Definition 1 of Limit, **Alternate Definition 2 of Limit**, Properties of Limits (Theorem 1—especially note the Root Rule), computations of limits, continuity, Two-Path Test for Nonexistence of a Limit, Continuity of Compositions of Functions.

**14.3 Partial Derivatives.** Partial derivatives with respect to  $x$  and  $y$  (and  $z$ ), higher order partial derivatives, **The Mixed Derivative Theorem** (Theorem 2, "Clairaut's Theorem"), definition of  $f(x, y)$  is differentiable at  $(x_0, y_0)$ , Differentiability Implies Continuity (Theorem 4).

**14.4 The Chain Rule.** The Chain Rule for Functions of Two Independent Variables (Theorem 5), The Chain Rule for Functions of Three Independent Variables (Theorem 6), The Chain Rule for Functions of Two Independent Variables and Three Intermediate Variables (Theorem 7), branch diagrams for the Chain Rule, implicit differentiation and Theorem 8.

**14.5 Directional Derivatives and Gradient Vectors.** **Definition of directional derivative of  $f(x, y)$  at point  $P_0(x_0, y_0)$  in the direction  $\mathbf{u}$** , gradient vector, computation of directional derivatives using Theorem 9, Properties of Directional Derivatives, gradient is normal to level curves, tangent lines to normal curves, Algebraic Rules for Gradients.

**14.6 Tangents and Differentials.** Tangent planes and normal lines to a surface, **linearization of  $f(x, y)$  at point  $(x_0, y_0)$** , error approximation of linearization, differentials and total differential.

**14.7 Extreme Values and Saddle Points.** **Local maximum/local minimum**, First Derivative Test for Local Extreme Values (Theorem 10), **critical point**, **saddle point**, Second Derivative Test for Local Extreme Values (Theorem 11), discriminant/Hessian (curvature), finding absolute extrema on closed and bounded regions, method of least squares and regression line (page 828, number 65).

**14.8 Lagrange Multipliers.** Lagrange multipliers, **the Orthogonal Gradient Theorem**, the method of Lagrange multipliers, Lagrange multipliers with two constraints.

**14.9 Taylor's Formula for Two Variables.** Derivation of the Second Derivative Test, Derivation of the Error Formula for Linear Approximations, Taylor's Formula for  $f(x, y)$  at the Origin.

**14.10 Partial Derivatives with Constrained Variables.** Finding  $\partial w / \partial x$  when the variables in  $w = f(x, y, z)$  are constrained by another equation.