Chapter 11. Parametric Equations and Polar Coordinates

11.1. Parametrizations of Plane Curves

Definition. If x and y are given as functions $x = f(t)$ and $y = g(t)$ over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a *Parametric curve*. The equations are *parametric equations* and t is the *parameter*. If $I = [a, b]$ then $(f(a), g(a))$ is the *initial point* and $(f(b), g(b))$ is the *terminal point*.

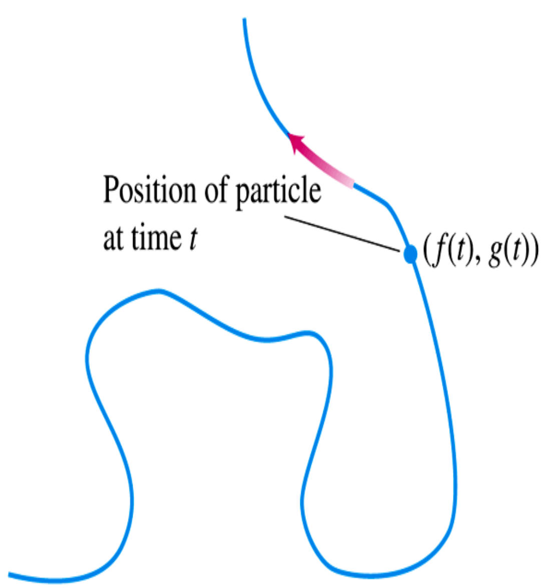

Figure 11.1, page 628

Examples. Page 629 Example 3(a), page 634 number 14, page 635 number 32, page 632 Example 8 (a cycloid).

Note. The cycloid (turned over from the way it is presented in Example 8) is a solution to the *brachistochrone problem* which involves determining the path along which a bead will slip along a frictionless wire from one point to another in a minimum amount of time. Though this sounds like a simple max/min problem, it requires an area of math called the *calculus of variations* to show that the cycloid is a solution to the brachistochrone problem (and the only solution). In fact, it takes the same amount of time for the bead to travel from any point P on the cycloid to the point B as given in the following figure. This makes the cycloid a *tautochrone*, or same-time curve. Such a curve was used by Christian Huygens in the construction of a pendulum driven clock.

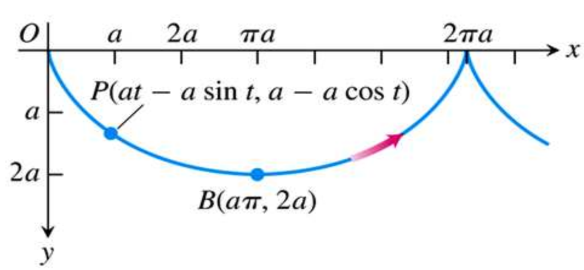

Figure 11.10, page 632

Example. Page 635 number 36.