Chapter 11. Parametric Equations and Polar Coordinates

11.3. Polar Coordinates

Definition. We define the *polar coordinates* of a point $P(r, \theta)$ in the Cartesian plane by introducing an *initial ray* from the origin O which lies along the x-axis. Point P is then said to lie at $P(r, \theta)$ if either (1) it lies a distance r ($r \geq 0$) along a ray which makes an angle of θ with the initial ray, or (2) it lies a distance $-r$ ($r \leq 0$) along a ray which makes an angle of θ with the initial ray. Coordinate r gives the *directed distance* of P from O.

Note. Due to the the fact that coterminal rays can be represented with different values of θ, then a point in the Cartesian plane can have multiple representations in polar coordinates.

Example. Page 648, number 4c.

Note. If we hold r fixed at a constant value, $r = a \neq 0$, then the point $P(r, \theta)$ will lie $|a|$ units from the origin O. As θ varies over any interval of length 2π, P then traces a circle of radius $|a|$ centered at O. If we hold θ fixed at a constant value $\theta = \theta_0$ and let r vary between $-\infty$ and ∞, the point $P(r, \theta)$ traces the line through O that makes an angle of measure θ_0 with the initial ray. In general, we can relate Cartesian (x, y) coordinates to polar coordinates $P(r, \theta)$ as:

$$x = r\cos\theta,\ y = r\sin\theta,\ r^2 = x^2 + y^2,\ \tan\theta = \frac{y}{x}.$$

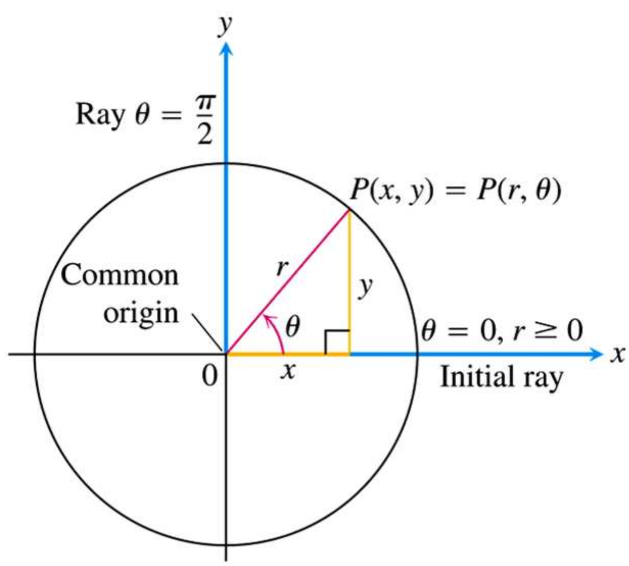

Figure 11.24, page 647

Examples. Page 649, numbers 36 and 62.