Chapter 15. Multiple Integrals

15.3. Area by Double Integration

Note. If we take $f(x, y) = 1$ in the definition of the double integral over a region R in the preceding section, the Riemann sums reduce to

$$S_n = \sum_{k=1}^{n} f(x_k, y_k) \Delta A_k = \sum_{k=1}^{n} \Delta A_k.$$

This is simply the sum of the areas of the small rectangles in the partition of R, and approximates what we would like to call the area of R.

Definition. The *area* of a closed, bounded plane region R is

$$A = \iint_R dA.$$

Examples. Page 870, numbers 8 and 14.

Definition. The *average value* of $f(x, y)$ over region R is

$$\frac{1}{\text{area of } R} \iint_R f \, dA.$$

Example. Page 870, number 20.

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