Problem 2.38

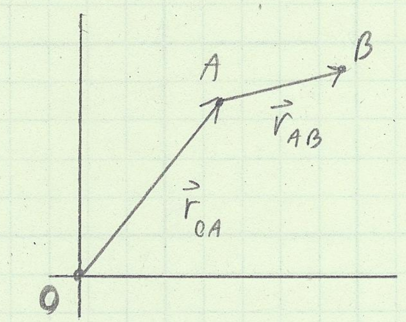
<u>Given</u>: $\vec{r}_{OA} = 400\,\hat{\imath} + 800\,\hat{\jmath}$ (m)

$|\vec{r}_{AB}| = 400$ m

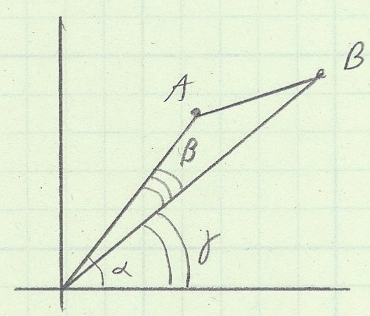
$|\vec{r}_{OA} + \vec{r}_{AB}| = |\vec{r}_{OB}| = 1200$ m

<u>FIND</u>: The coordinates of B.

<u>Solution</u>: Free body diagram (FBD):



Our strategy is to find α using right triangles, β using the Law of Cosines and γ from the fact that $\gamma = \alpha - \beta$:



The coordinates of β then follow easily since we know $|\vec{r}_{OB}| = |\vec{r}_{OA} + \vec{r}_{AB}|$. Since $A = (400, 800)$ (m), then $\tan(\alpha) = \dfrac{800\text{ m}}{400\text{ m}} = 2$ and $\alpha = 63.43°$ (since α is a first quadrant angle). From the Law of Cosines,

$$|\vec{r}_{AB}|^2 = |\vec{r}_{OA}|^2 + |\vec{r}_{OB}|^2 - 2|\vec{r}_{OA}|\,|\vec{r}_{OB}|\cos(\beta).$$

That is, $\cos(\beta) = \dfrac{|\vec{r}_{AB}|^2 - |\vec{r}_{OA}|^2 - |\vec{r}_{OB}|^2}{-2|\vec{r}_{OA}|\,|\vec{r}_{OB}|}$.

Therefore if β is acute (as the picture suggests) then

$$\beta = \cos^{-1}\left(\frac{|\vec{r}_{AB}|^2 - |\vec{r}_{OA}|^2 - |\vec{r}_{OB}|^2}{-2\,|\vec{r}_{OA}|\,|\vec{r}_{OB}|}\right)$$

$$= \cos^{-1}\left(\frac{(400\text{m})^2 - ((400\text{m})^2 + (800\text{m})^2) - (1200\text{m})^2}{-2\sqrt{(400\text{m})^2 + (800\text{m})^2}\,(1200\text{m})}\right)$$

$$= 14.31^\circ.$$

(Notice that $\beta = -14.31^\circ$ is also admissible.)

Since $\alpha = 63.43^\circ$, we have

$$\gamma = \alpha - \beta = 63.43^\circ - (14.31^\circ) = 49.12^\circ.$$

Finally, the x-coordinate of B is

$$|\vec{r}_{OB}|\cos(49.12^\circ) = 785\text{ m}$$

and the y-coordinate of B is

$$|\vec{r}_{OB}|\sin(49.12^\circ) = 907\text{ m}.$$

Final Answer: $\boxed{B = (785, 907)\ (\text{m})}$

NOTE: If we take $\beta = -14.31^\circ$, then we get

$$\boxed{B = (255, 1173)\ (\text{m})}$$

by similar computations.