Problem 3.79

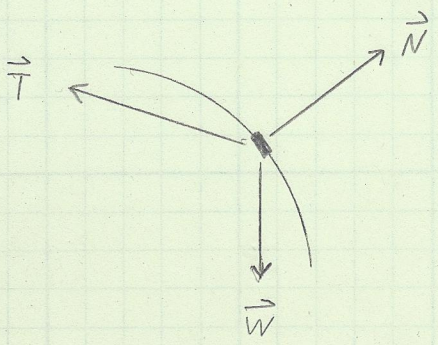
Given: weight $\vec{w} = -100\hat{j}$ lb
circular bar
information in figure

Find: (a) tension in cable AB.
(b) normal force exerted on the slider by the bar.

Soln: Since the bar is a semicircle of radius 4 ft, the coordinates of point A are
$(r\cos(20°), r\sin(20°), 0) = (3.759, 1.368, 0)$ (ft).
The position vector for point A is then
$\vec{r}_A = 3.759\hat{i} + 1.368\hat{j}$ and so a unit vector parallel to the bar at point A is

$$\hat{e}_p = \frac{-1.368\hat{i} + 3.759\hat{j}}{\sqrt{(-1.368)^2 + (3.759)^2}} = -.342\hat{i} + .940\hat{j}.$$

At point A we have three forces: $\vec{w} = -100\hat{j}$ lb, $\vec{T}$ the tension in cable AB, and $\vec{N}$ is the force the bar exerts on the slider. The FBD at A is



The equilibrium equation at A is

$$\sum \vec{F} = \vec{T} + \vec{N} + \vec{W} = \vec{0}.$$

Since $\hat{e}_p$ is perpendicular to $\vec{N}$, we have

$$\hat{e}_p \cdot \left(\sum \vec{F}\right) = \hat{e}_p \cdot \vec{T} + \hat{e}_p \cdot \vec{N} + \hat{e}_p \cdot \vec{W}$$

$$= \hat{e}_p \cdot \vec{T} + \hat{e}_p \cdot \vec{W} = 0.$$

Now $\hat{e}_p \cdot \vec{W} = (-.342\,\hat{i} + .940\,\hat{j}) \cdot (-100\,\hat{j})$ (lb)

$= -94$ lb.

A unit vector along cable AB is

$$\hat{e}_{AB} = \frac{(0-3.759)\,\hat{i} + (4-1.368)\,\hat{j} + (3-0)\,\hat{k}}{\sqrt{(3.759)^2 + (2.632)^2 + (3)^2}}$$

$$= -.686\,\hat{i} + .480\,\hat{j} + .547\,\hat{k}.$$

Then the tension in the cable is $\vec{T} = |\vec{T}|\,\hat{e}_{AB}$.

Now $\hat{e}_p \cdot \vec{T} = |\vec{T}|\left((-.342)(-.686) + (.940)(.480) + (0)(.547)\right)$

$= .686\,|\vec{T}|$.

Therefore $\hat{e}_p \cdot \vec{T} + \hat{e}_p \cdot \vec{W} = 0$

implies $.686\,|\vec{T}| + (-94\text{ lb}) = 0$

and so $|\vec{T}| = 137$ lb,

and $\vec{T} = -94\,\hat{i} + 65.8\,\hat{j} + 74.9\,\hat{k}$ (lb).

From the equilibrium equation,

$$\vec{N} = -\vec{T} - \vec{W}$$

$$= -(-94\hat{i} + 65.8\hat{j} + 74.9\hat{k}) - (-100\hat{j}) \quad (lb)$$

$$= 94\hat{i} + 34.2\hat{j} + 74.9\hat{k} \quad (lb).$$

Final Answer:

(a) $\vec{T} = -94\hat{i} + 65.8\hat{j} + 74.9\hat{k}$ (lb)

(b) $\vec{N} = 94\hat{i} + 34.2\hat{j} + 74.9\hat{k}$ (lb)