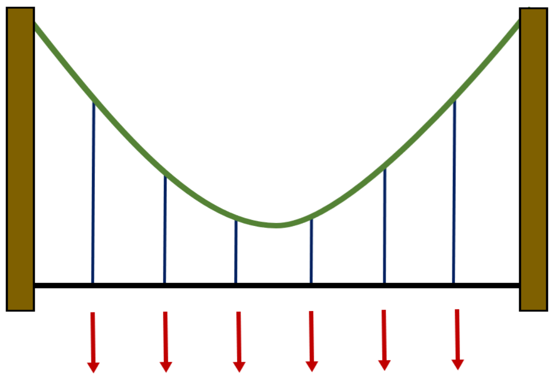
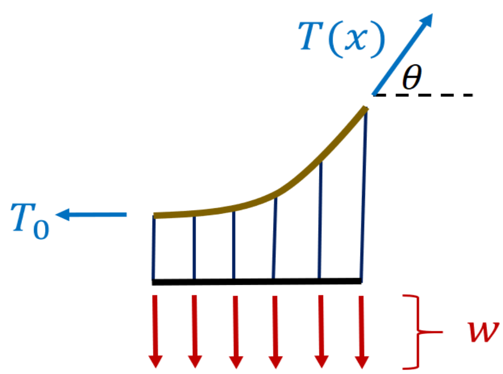
Section 10.4. Loads Distributed Uniformly Along Straight Lines

Note. Consider a suspension bridge. The load on the suspension cable is assumed to be uniform (this represents the road and roadbed):



We cut the cable at its lowest point and at a point a distance x from the lowest point:



Let T_0 be the tension at the lowest point and let $T(x)$ be as in the picture. Then from the example equations:

$$\left.\begin{aligned} T_0 &= T(x)\cos\theta \\ wx &= T(x)\sin\theta. \end{aligned}\right\} \qquad (*)$$

Then $\tan\theta = wx/T_0 = ax$ where $a = wx/T_0$. Since $\tan\theta$ is the slope of the cable at a given point,

$$\frac{dy}{dx} = \tan\theta = ax. \qquad (**)$$

Therefore $y = \frac{a}{2}x^2 + C$. By convention, we assume the origin occurs at the lowest point and so $y(0) = 0$, which implies $C = 0$ and $y = \frac{1}{2}ax^2$ where $a = wx/T_0$.

Note. Since from $(*)$

$$T_0^2 = T(x)\cos^2\theta$$
$$w^2x^2 = T(x)^2\sin^2\theta$$

and so

$$T(x)^2 = T_0^2 + w^2x^2 = T_0^2 + T_0^2a^2x^2 = T_0^2(1 + a^2x^2)$$

or $T(x) = T_0\sqrt{1 + a^2x^2}$.

Note. The length of the cable over the interval 0 to x is

$$\begin{aligned}\int_0^x \sqrt{1 + (dy/dx)^2}\,dx &= \int_0^x \sqrt{1 + (ax)^2}\,dx \text{ from } (**)\\ &= \int_0^x \sqrt{1 + (au)^2}\,du \text{ use trig substitution with } u = \frac{1}{a}\sec\theta\ldots\\ &= \frac{1}{2}\left\{x\sqrt{1 + a^2x^2} + \frac{1}{a}\ln\left(ax + \sqrt{1 + a^2x^2}\right)\right\}\end{aligned}$$

Example. Page 526 Number 10.55.

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