Section 2.4. Components in Three Dimensions

Note. We introduce 3 dimensional vectors in a right hand coordinate system (see page 42). We denote a unit vector in the positive x direction as $\hat{\imath}$, a unit vector in the positive y direction as $\hat{\jmath}$ and a unit vector in the positive z direction as $\vec{k}$. We can therefore write any 3-dimensional vector $\vec{u}$ as $\vec{u}_x + \vec{u}_y + \vec{u}_z$ where $\vec{u}_x$ is a multiple of $\vec{\imath}$, $\vec{u}_y$ is a multiple of $\hat{\jmath}$, and $\vec{u}_z$ is a multiple of $\vec{k}$. In terms of components, $\vec{u} = u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}$.

Note. From the Pythagorean Theorem,

$$\|\vec{u}| = |u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}| = \sqrt{(u_x)^2 + (u_y)^2 + (u_z)^2}.$$

Definition. Define the *direction cosines* of $\vec{u} = u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}$ as

$$\cos\theta + x = \frac{u_x}{|\vec{u}|},\ \cos\theta_y - \frac{u_y}{|\vec{u}|},\ \cos\theta_z = \frac{u_z}{|\vec{u}|}.$$

Note. The direction cosines of vectors $\vec{u}$ are the components of a unit vector with the same direction as $\vec{u}$. Therefore

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1.$$

Note. The vector from point $A = (x_A, y_A, z_A)$ to point $B = (x_B, y_B, z_B)$ is

$$\vec{r}_{AB} = (x_B - z_A)\hat{\imath} + (y_B - y_A)\hat{\jmath} + (z_B - z_A)\hat{k}.$$

Examples. Page 53 Numbers 2.79 and 2.90.

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