Section 2.5. Dot Products

Definition. The *dot product* of vectors $\vec{u}$ and $\vec{v}$ is a scalar

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$$

where θ is the angle between $\vec{u}$ and $\vec{v}$.

Note. The dot product satisfies:

1. $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$,

2. $a(\vec{u} \cdot \vec{v}) = (a\vec{u}) \cdot \vec{v} = \vec{u} \cdot (a\vec{v})$, and

3. $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$.

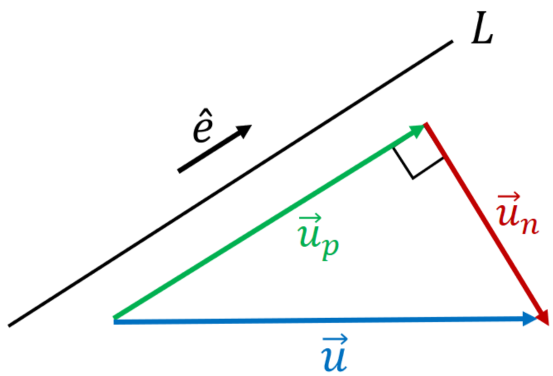
Note. Since $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are mutually orthogonal:

$$\begin{array}{rrr} \hat{\imath} \cdot \hat{\imath} = 1 & \hat{\imath} \cdot \hat{\jmath} = 0 & \hat{\imath} \cdot \hat{k} = 0 \\ & \hat{\jmath} \cdot \hat{\jmath} = 1 & \hat{\jmath} \cdot \hat{k} = 0 \\ & & \hat{k} \cdot \hat{k} = 1 \end{array}$$

From these properties, it follows that

$$\vec{u} \cdot \vec{v} = (u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}) \cdot (v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}) = u_xv_x + u_yv_y + u_zv_z.$$

Note. We break a vector into components parallel to and perpendicular to a given line. Let $\hat{e}$ be a unit vector in the direction of line L. The vector $\vec{u}$ can be decomposed as follows:



Example. Page 66 Number 2.121. HINT: Find point A using vectors, find $\vec{T}$ and project onto DC.

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