Section 2.6. Cross Products

Definition. For two 3-dimensional vectors $\vec{u}$ and $\vec{v}$, define the *cross product* of $\vec{u}$ and $\vec{v}$ as

$$\vec{u} \times \vec{v} = |\vec{u}||\vec{v}|(\sin\theta)\hat{e}$$

where $\vec{e}$ is a unit vector perpendicular to both $\vec{u}$ and $\vec{v}$ determined by a right hand rule by curling the fingers of the right had from $\vec{u}$ to $\vec{v}$ and then the $\hat{e}$ vector is determined from the location of the thumb. Notice that if $\vec{u}$ and $\vec{v}$ are parallel (or antiparallel) then $\vec{u} \times \vec{v} = \vec{0}$.

Note. Some Properties of cross product are:

1. $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$,

2. $a(\vec{u} \times \vec{v}) = (a\vec{u}) \times \vec{v} = \vec{u} \times (a\vec{v})$, and

3. $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$.

Note. We find:

$$\begin{array}{ccc} \hat{\imath} \times \hat{\imath} = \vec{0} & \hat{\imath} \times \hat{\jmath} = \hat{k} & \hat{\imath} \times \hat{k} = -\hat{\jmath} \\ \hat{\jmath} \times \hat{\imath} = -\hat{k} & \hat{\jmath} \times \hat{\jmath} = \vec{0} & \hat{\jmath} \times \hat{k} = \hat{\imath} \\ \hat{k} \times \hat{\imath} = \hat{\jmath} & \hat{k} \times \hat{\jmath} = -\hat{\imath} & \hat{k} \times \hat{k} = \hat{0}. \end{array}$$

Note. In terms of components, $\vec{u} \times \vec{v}$ is calculated as

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_x & u_y & u_z \\ v_z & z_y & v_z \end{vmatrix} = \begin{vmatrix} u_y & u_z \\ v_y & v_z \end{vmatrix} \hat{\imath} - \begin{vmatrix} u_x & u_z \\ v_x & v_z \end{vmatrix} \hat{\jmath} + \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} \hat{k}$$

$$= (u_y v_z - u_z v_y)\hat{\imath} - (u_x v_z - u_z v_x)\hat{\jmath} + (u_x v_y - u_y v_x)\hat{k}.$$

Example. Page 76 Number 2.140.

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