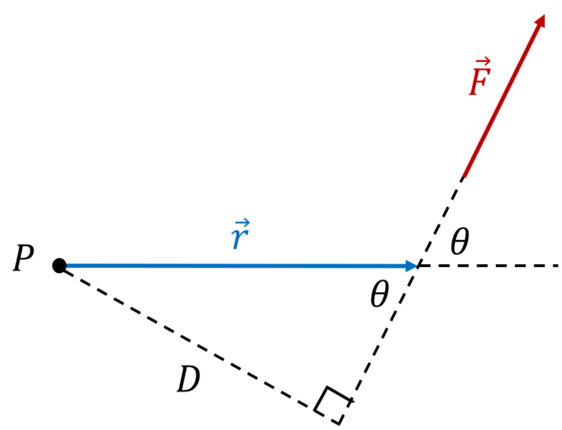
Section 4.2. The Moment Vector

Definition. Consider a force $\vec{F}$ (with a line of action) and a point P. The *moment* of $\vec{F}$ about point P is $\vec{M}_P = \vec{r} \times \vec{F}$ where $\vec{r}$ is a position vector from P to any point on the line of action of $\vec{F}$.

Note. $|\vec{M}_P| = |\vec{r}||\vec{F}| \sin\theta$ where θ is the angle between $\vec{r}$ and $\vec{F}$. The perpendicular distance from P to the line of action of $\vec{F}$ is $D = |\vec{r}| \sin\theta$, so $|\vec{M}_P| = D|\vec{F}|$ (as in Section 4.1):



Note. If $\vec{r}$ and $\vec{r}'$ are vectors from P to the line of action of $\vec{F}$, then $\vec{r} = \vec{r}' + \vec{u}$ where $\vec{u}$ is a vector along the line of action of $\vec{F}$ and

$$\vec{r} \times \vec{F} = \vec{r} \times \vec{F} + \underbrace{\vec{u} \times \vec{F}}_{\vec{0}} = (\vec{r} + \vec{u}) \times \vec{F} = \vec{r}' \times \vec{F}.$$

Note. The positive or negative sense of a moment is reflected in the right hand rule for $\vec{r} \times \vec{F}$ (see page 139).

Theorem (Varignon's). Let $\vec{F}_1, \vec{F}_2, \ldots, \vec{F}_N$ be a system of forces whose lines of action intersect at point Q. Then the moment of th esystem about a point P is

$$(\vec{r}_{PQ} \times \vec{F}_1) + (\vec{r}_{PQ} \times \vec{F}_2) + \cdots + (\vec{r}_{PQ} \times \vec{F}_N) = \vec{r}_{PQ} \times (\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N).$$

Example. Page 150 Number 4.69.

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