Chapter 8. Moments of Inertia

Note. Recall that a particle of mass m with velocity v has kinetic energy $\frac{1}{2}mv^2$.

Note. This note is based on Section 15.6, "Moments and Centers of Mass," of *Thomas Calculus*, 12th edition. See also my online notes:

`http://faculty.etsu.edu/gardnerr/2110/notes-12e/c15s6.pdf`.

If a rod rotates around its axis with angular velocity ω, then we can associate a kinetic energy with a differential of mass dm a distance r from the axis of

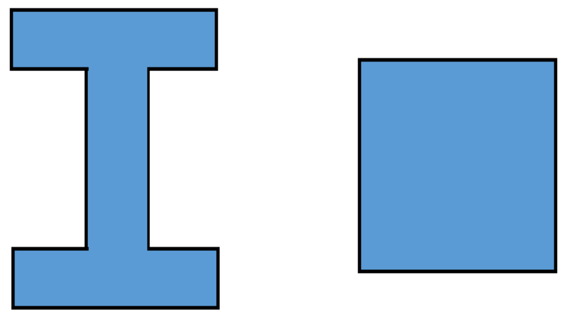
$$\frac{1}{2}(dm)v^2 = \frac{1}{2}(dm)(\omega r)^2 = \frac{\omega^2}{2}r^2\,dm.$$

The total kinetic energy of the rod is then:

$$\int_M \frac{\omega^2}{2}r^2\,dm = \frac{\omega^2}{2}\int_M r^2\,dm.$$

If we define $I = \int_M r^2\,dm$ then the kinetic energy is $\frac{1}{2}I\omega^2$. I is the *moment of inertia* of the rod and reflects how hard it is to start the rod rotating (or to stop it from rotating once it has started).

Note. Beams are often "I beams" because they have a greater moment of inertia and therefore are harder to bend:



With the same cross sectional area, the I beam is stronger.

Similarly, tubes are stiffer than rods.

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