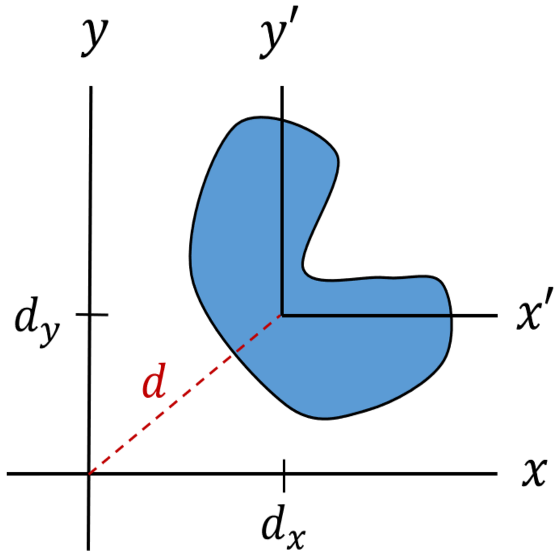
Section 8.2. Parallel-Axis Theorem

Note. Consider a set of coordinates x', y' where the origin of this coordinate system lies at the centroid of an area A_0 Let there be another coordinate system, x, y, as follows:



In the x', y' coordinates, the centroid of the area is

$$\overline{x}' = \frac{\int_A x'\,dA}{\int_A dA} \text{ and } \overline{y}' = \frac{\int_A y'\,dA}{\int_A dA}$$

and so

$$\overline{x}' = \int_A x'\,dA = \overline{y}' = \int_A y' = 0.$$

In the x, y coordinate system the moment of inertia about the x-axis is $I_x = \int_A y^2\,dA$. With the above notation,

$$I_x = \underbrace{\int_A (y' + dy)^2\,dA}_{I_{x'}} = \int_A (y')^2\,dA + 2dy\int_A y'\,dA + dy^2\int_A dA.$$

As above, $\overline{y}' = \int_A y'\,dA = 0$ and so

$$I_x = I_{x'} + dy = \int_A dA.$$

Theorem. (Parallel-Axis Theorem). With the above notation:

$$I_x = I_{x'} + dy^2 A, \;\; I_y = I_{y'} + dx^2 A, \;\; I_{xy} = I_{x'y'} + dx\, dy\, A, \;\; J_0 = J_{0'} + d^2 A.$$

Examples. Page 405 Numbers 8.41 and 8.44.

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