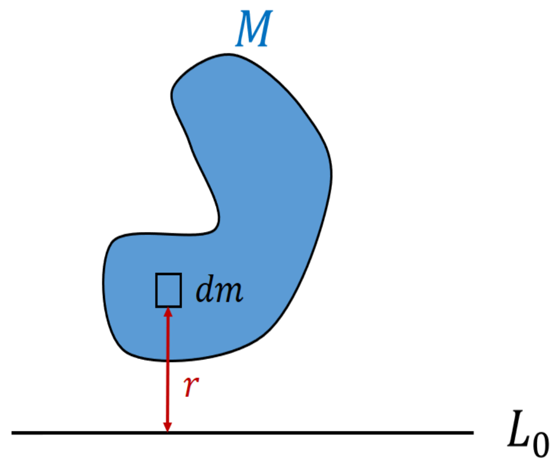
Section 8.4. Simple Objects

Note. The *mass moment of inertia* about line (or "axis") L_0 is $I_0 = \int_M r^2\, dm$ where r is the perpendicular distance from the axis to the differential element of mass dm:



Note. As always, we can calculate the mass moments of inertia of a complicated object by summing the mass moments of inertia of the simple objects which compose it.

Note. Suppose we have a bar of cross sectional area A, length ℓ and mass m. Let density ρ satisfy $dm = \rho A\, dr$. If the bar is infinitely thin (!) then the mass moment of inertia about a line through the center of mass and perpendicular to the bar is

$$I = \int_M r^2\, dm = \int_{-\ell/2}^{\ell/2} \rho A r^2\, dr = \frac{1}{12}\rho A \ell^3 = \frac{1}{12} m \ell^2$$

since $m = \rho A \ell$.

Note. Consider a thin plate P in the xy plane of thickness T and uniform density ρ. Let dm be a differential of mass a distance r from the origin. The *mass moment of inertia* about the z-axis is:

$$I_{(z\text{ axis})} = \int_P r^2\,dm = \rho T \int_P r^2\,dA = \frac{m}{A} J_0$$

since $m = \rho T A$. The mass moment of inertia about the x-axis is

$$I_{(x\text{ axis})} = \int_P y^2\,dm = \rho T \int_P y^2\,dA = \frac{m}{A} I_x.$$

The mass moment of inertia about the y-axis is

$$I_{(y\text{ axis})} = \int_P x^2\,dm = \rho T \int_P x^2\,dA = \frac{m}{A} I_y.$$

Notice $I_{(z\text{ axis})} = I_{(x\text{ axis})} + I_{(y\text{ axis})}$.

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