# **Section 1.2.** Propositional Equivalences

**Note.** In this section we consider propositions with the same truth values.

**Definition 1.2.1.** A compound proposition that is always true is a *tautology*. A compound proposition that is always false is a *contradiction*. A proposition that is neither a tautology nor a contradiction is a *contingency*.

**Definition 1.2.2.** The propositions $p$ and $q$ are *logically equivalent* if $p \leftrightarrow q$ is a tautology. this is denoted $p \Leftrightarrow q$. (Logically equivalent propositions have the same truth tables.)

**Example.** Example 1.2.2 Page 15. DeMorgan's Law states that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent. The truth table is:

| $p$ | $q$ | $\neg p$ | $\neg q$ | $p \vee q$ | $\neg(p \vee q)$ | $\neg p \wedge \neg q$ |
|---|---|---|---|---|---|---|
| $T$ | $T$ | $F$ | $F$ | $T$ | $F$ | $F$ |
| $T$ | $F$ | $F$ | $T$ | $F$ | $T$ | $T$ |
| $F$ | $T$ | $T$ | $F$ | $F$ | $T$ | $T$ |
| $F$ | $F$ | $T$ | $T$ | $F$ | $T$ | $T$ |

**Examples.** Number 6, Page 19, Number 20 Page 16, Numbers 30 and 32 Page 20, Number 26 Page 26.

**Note.** For each line in the truth table we can take the conjugation of each of the $T$ values along with the conjugation of the negation of each of the $F$ variables. Then this compound proposition is only $T$ for the line in the truth table from which the proposition was constructed. By taking the disjunction over all lines in the table for which the proposition is $T$, we get the proposition with the given truth table in *disjunctive normal form*. For example:

| $p$ | $q$ | $r$ | |
|---|---|---|---|
| $T$ | $T$ | $F$ | |
| $T$ | $F$ | $T$ | $p \wedge \neg q$ |
| $F$ | $T$ | $T$ | $\neg p \wedge q$ |
| $F$ | $F$ | $T$ | $\neg p \wedge \neg q$ |

.

Then $r$ can be represented as

$$(p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$$

$r$ is then only $F$ when each of the three "little" propositions are $F$.

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