Section 1.3. Predicates and Quantifiers

Note. In this section we increase our vocabulary.

Note. Consider the statement "x is greater than 3." The "x" is the *subject* of the statement and "is greater than 3" is the *predicate* of the statement. If we denote the statement by $P(x)$ then P is called the *propositional function*. We then have $P(4)$ is T and $P(2)$ is F (for example). We can also have propositional functions of several variables: $R(x, y, z)$ can represent "$x + y = z$" in which case $R(1, 1, 2)$ is T and $R(1, 1, 3)$ is F.

Definition. When dealing with a propositional function, the set of all possible values of the variables is called the *universe of discourse* of the proposition.

Definition 1.3.1. The *universal quantification* of $P(x)$ is the proposition "$P(x)$ is true for all values of x in the universe of discourse." This is denoted $\forall x P(x)$.

Definition 1.3.2. The *existential quantification* of $P(x)$ is the proposition "There exists an element x in the universe of discourse such that $P(x)$ is true." This is denoted $\exists x P(x)$.

Example. Example 19 Page 28.

Definition. When a quantifier is used on a variable x or when we assign a value to this variable, we say that this occurrence of the variable is a *bound variable.* Otherwise the variable is a *free variable.*

Note. On pages 32 and 33, the book sates some equivalent propositions:

$$\neg \forall x P(x) \Leftrightarrow \exists x \neg P(x)$$

$$\neg \exists x Q(x) \Leftrightarrow \forall x \neg Q(x).$$

Examples. Page 33 Numbers 4, 8b, 8d, 16a, 16b, 20, 44, 52.

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