Section 1.4. Sets

Note. In this section we study naive set theory, as opposed to axiomatic set theory (see page 38). For a consideration of axiomatic set theory, see my online notes at `http://faculty.etsu.edu/gardnerr/Set-Theory-Intro/notes.htm`

Definition 1.4.1. The objects in a set are also called the *elements* of the set. A set is said to *contain* its elements.

Definition 1.4.2. Two sets are *equal* if and only if they have the same elements.

Note. We can build sets from existing sets using *set builder* notation:

$$\{x \mid x \text{ is an odd integer between 0 and 10}\} = \{1, 3, 5, 7, 9\}.$$

Example. Page 45 Number 2.

Definition 1.4.3. The set A is said to be a *subset* of B if and only if every element of A is also an element of B. This is denoted $A \subseteq B$. If $A \subseteq B$ but $A \neq B$ then A is a *proper subset* of B, denoted $A \subset B$. (WARNING: Some texts us the notation $A \subset B$ in place of $A \subseteq B$.)

Example. Page 45 Number 8.

Definition 1.4.4. Let S be a set. If there are exactly n distinct elements in S where n is a nonnegative integer, then S is a *finite set* and the *cardinality* of S is n, denoted $|S| = n$.

Definition 1.4.5. A set is *infinite* if it is not finite.

Definition 1.4.6. The *power set* of a set S is the set of all subsets of S, denoted $\mathcal{P}(S)$.

Note. If $|S| = n$ then $|\mathcal{P}(S)| = 2^n$.

Definition 1.4.7. The *ordered n-tuple* $(a_1, a_2, \ldots, a_n)$ is the ordered collection with a_1 first, a_2 second, etc.

Definition 1.4.8. Let A and B be sets. The *Cartesian product* of A and B, denoted $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.

Definition 1.4.9. The *Cartesian product* of sets $A_1, A_2, \ldots, A_n$, denoted $A_1 \times A_2 \times \cdots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots, a_n)$ where $a_i \in A_i$.

Examples. Page 45 Numbers 22 and 26.

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