Section 1.6. Functions

Note. In this section we define functions form one set to another and state several definitions describing such functions.

Definition 1.6.1. Let A and B be sets. A *function* f from A to B is an assignment of exactly one element of B to each element of A. We write $f(a) = b$ if b is the element of B assigned by f to element a of A. We also write $f : A \to B$.

Definition 1.6.2. If $f : A \to B$, then A is the *domain* of f and B is the *codomain* of f. If $f(a) = b$, then b is the *image* of a and a is the *pre-image* of b. The *range* of f is the set of all images of elements of A. We also say f *maps* A to B.

Example. Page 67 Number 2.

Definition 1.6.3. Let f_1 and f_2 be functions from A to $\mathbb{R}$. Then define $f_1 + f_2$ and $f_1 f_2$ as $(f_1 + f_2)(x) = f_1(x) + f_2(x)$ and $(f_1 f_2)(x) = f_1(x) f_2(x)$.

Definition 1.6.4. Let $f : A \to B$, $S \subset A$. The *image* of set S is $f(S) = \{f(s) \mid s \in S\}$.

Definition 1.6.5. A function f is *one-to-one* (or *injective*) if and only if $f(x) = f(y)$ implies $x = y$. f is called an *injection*.

Note. The contrapositive of Definition 1.6.5 is: $x \neq y \Rightarrow f(x) \neq f(y)$.

Example. Page 67 Numbers 8 and 12.

Definition 1.6.6. Suppose the domain and codomain of f are subsets of $\mathbb{R}$. Then f is *strictly increasing* if $f(x) < f(y)$ whenever $x < y$ and x, y are in the domain of f. f is *strictly decreasing* if $f(x) > f(y)$ whenever $x < y$ and x, y are in the domain of f.

Definition 1.6.7. A function $f : A \to B$ is *onto* (or *surjective*) if and only if every element of B is the image of an element of A. That is $f(A) = B$. f is called a *surjection.*

Definition 1.6.8. Function f is a *bijection* if it is both one to one and onto.

Definition 1.6.9. Let $f : A \to B$ be a bijection. The *inverse function* of $f$is defined as $f^{-1}(b) = a$ if and only if $f(a) = b$.

Examples. Page 68 Number 30.

Definition 1.6.10. Let $g : A \to B$ and $f : B \to C$. The *composition* of f and g, denoted $f \circ g$, is defined as $(f \circ g)(a) = f(g(a))$.

Definition 1.6.11. Let $f : A \to B$. The *graph* of f is the set of ordered pairs

$$\{(a, b) \mid a \in A \text{ and } f(a) = b\}.$$

Definition 1.6.12. The *floor function* assigns to the real number x the largest integer that is less than or equal to x, denoted $\lfloor x \rfloor$. The *ceiling function* assigns to real number x the smallest integer that is greater than or equal to x, denoted $\lceil x \rceil$.

Example. Page 69 Number 54a.

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