Section 1.7. Sequences and Summations

Note. In this section we consider sequences and their (finite) sums. We also deal briefly with the cardinality of an infinite set.

Definition 1.7.1. A *sequence* is a function from a subset of the set of integers to a set S. The image of the integer n is denoted a_n (a *term* of the sequence).

Definition. An *arithmetic progression* is a sequence of the form $a, a+d, a+2d, a+3d, \ldots$.

Definition. We denote sums of elements of sequences as

$$\sum_{j=m}^{n} a_j = a_m + a_{m+1} + \cdots + a_n.$$

Example. Page 79 Number 16.

Definition. A *geometric progression* is a sequence of the form $a, ar, ar^2, ar^3, \ldots$.

Theorem. $\displaystyle\sum_{j=0}^{n} ar^j = \frac{ar^{n+1} - a}{r-1}$ for $r \neq 0$, $r \neq 1$.

Proof. Let $S = \sum_{j=0}^{n} ar^j$. Then $rS = \sum_{j=0}^{n} ar^{j+1}$ and $rS - S = ar^{n+1} - a$ or $S = \dfrac{ar^{n+1} - a}{r-1}$, as claimed. ∎

Note. We have:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

Definition 1.7.2. The sets A and B have the *same cardinality* if and only if there is a one to one correspondence from A to B.

Definition 1.7.3. A set that is either finite or has the same cardinality as the set of natural numbers is *countable*. A set that is not countable is uncountable.

Example. Page 77 Example 17. The set $[0, 1]$ is not countable.

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