Section 3.2. Mathematical Induction

Note. In this section we introduce the principle of mathematical induction which is a technique to show that a proposition holds for all positive integers.

Note. The following is an axiom for the integers.

> **Well-Ordering Property.** Every nonempty set of nonnegative integers has a least element.

Note. The method of *Mathematical Induction* follows two steps:

1. Basic Step. The proposition $P(1)$ is shown to be true.

2. Inductive Step. The implication $P(n) \to P(n+1)$ is shown to be true for every $n \in \mathbb{Z}^+$.

After these steps have been shown, it follows that $P(n)$ holds for all $n \in \mathbb{Z}^+$. Symbolically,

$$[P(1) \land \forall n(P(n) \to P(n+1))[\to \forall n P(n).$$

Example. Page 200 Number 14.

Note. The *Second Principle of Mathematical Induction* follows two steps:

1. Basic Step. The proposition $P(1)$ is shown true.

2. Inductive Step. The implication $[P(1) \wedge P(2) \wedge \cdots \wedge P(n)] \rightarrow P(n+1)$ is shown to be true for every $n \in \mathbb{Z}^+$.

These two steps show $\forall n P(n)$.

Examples. Page 200 Number 20, Page 198 Example 14.

Revised: 4/4/2019