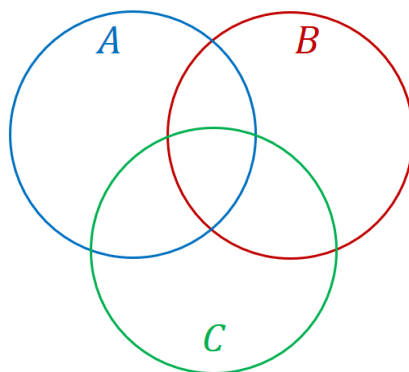# Section 5.5. Inclusion-Exclusion

Note. In this section we address cardinalities of unions and intersections of finite sets.

Note. We have seen that $|A \cup B| = |A| + |B| - |A \cap B|$.

Note. A Venn diagram for sets A, B, C is

and

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |C \cap C| + |A \cap B \cap C|.$$

Theorem 5.5.1. Principle of Inclusion-Exclusion.

Let $A_1, A_2, \ldots, A_n$ be finite sets. Then

$$|A_1 \cup A_2 \cup \cdots \cup A_n| = \sum_{i=1}^{n} |A_i| - \sum_{1 \le i < j \le n} |A_i \cap A_j|$$

$$+ \sum_{a \le i < j < k \le n} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n+1} |A_1 \cap A_2 \cap \cdots \cap A_n|.$$

Example. Page 359 Example 5.

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