Chapter 6. Relations

Section 6.1. Relations and Their Properties

Note. In this section we define binary relations (which you will see again in Introduction to Modern Algebra, MATH 4127/5127) and consider different types of such relations.

Definition 6.1.1. Let A and B be sets. A *binary relation from A to B* is a subset of $A \times B$. If $a \in A$ and $b \in B$ are related (i.e., (a, b) is in the binary relation) then we write aRb.

Example. Page 382 Number 1b.

Definition 6.1.2. A *relation* on the set A is a binary relation from A to A (i.e., s subset of $A \times A$).

Definition 6.1.3. A relation R on a set A is *reflexive* if $(z, z) \in R$ for every $a \in A$.

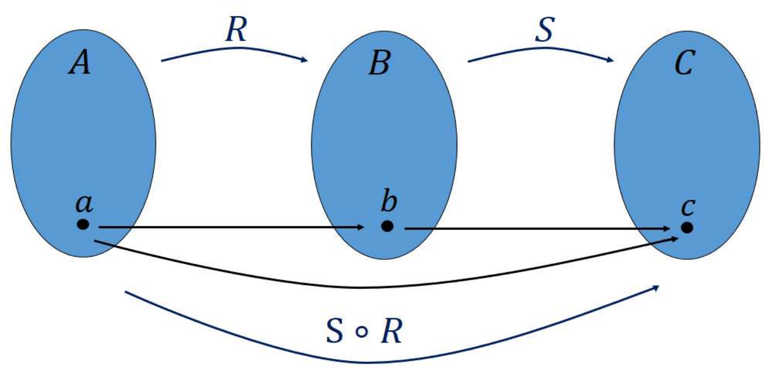
Definition 6.1.4. A relation R on a set A is *symmetric* if $(b, a) \in R$ whenever $(a, b) \in R$. A relation R such that both $(a, b) \in R$ and $(b, a) \in R$ only occurs when $a = b$, is called *antisymmetric*.

Example. $=$ on $\mathbb{R}$ is symmetric. $\leq$ on $\mathbb{R}$ is antisymmetric.

Definition 6.1.5. A relation R on a set A is *transitive* if $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$.

Example. $<$ on $\mathbb{R}$ is transitive.

Definition 6.1.6. Let R be a relation from A to B and let S be a relation from B to C. The *composite* of R and S is the relation of ordered pairs (a, c) where $a \in A$, $c \in C$, and for which $(a, b) \in R$ and $(b, c) \in S$ for some $b \in B$. This composite is denoted $S \circ R$.



Example. Page 383 Number 20.

Definition 6.1.7. Let R be a relation on A. Define $R^n = R \circ R \circ \cdots \circ R$ (n times).

Theorem 6.1.1. The relation R on A is transitive if and only if $R^n \subseteq R$ for all $n \in \mathbb{N}$.

Example. Page 383 Number 34.