Section 6.3. Representing Relations

Note. In this section we represent relations using matrices and digraphs.

Note. We can represent relations with 01 matrices. Define for relation R matrix $M_R = [m_{ij}]$ where

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$$

Example. Page 395 Number 2a.

Note. If R is a relation on a set then M_R is square an dM_R reflects properties of R as follows:

1. R is reflexive if and only if the main diagonal of M_R consists of 1's only.

2. R is symmetric if and only if M_R is symmetric with respect to its main diagonal, $M_R = (M_R)^T$.

3. R is antisymmetric if $m_{ij} = 1$, $i \neq j$, implies $m_{ji} = 0$.

Example. Page 395 Example 6.

Theorem. For relations R_1, R_2, R, and S with corresponding matrix representations M_{R_1}, M_{R_2}, M_R, and M_S respectively, we have:

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2},\ M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2},$$

$$M_{S \circ R} = M_R \odot M_S,\ M_{R^n} = M_R^{[n]}.$$

Example. Page 395 Number 8.

Definition 6.3.1. A *directed graph*, or *digraph*, consists of a set V of *vertices* together with a set E of ordered pairs of V called *arcs*. For arc (a, b) vertex a is the *initial vertex* and b is the *terminal vertex*. An arc from a to a is a *loop*.

Example. Page 396 Number 14.

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