Chapter 7. Graphs

Note. In this chapter we introduce an idea of central importance in the area of discrete mathematics. The term "graph" used here is not to be confused with the "graph of a function," which is an unrelated concept.

Section 7.1. Introduction to Graphs

Note. In this section we define various kinds of graphs.

Definition 7.1.1. A *simple graph* $G = (V, E)$ consists of a *vertex set* V and an *edge set* E of unordered pairs of distinct vertices.

Definition 7.1.2. A *multigraph* $G = (V, E)$ consists of a *vertex set* V and *edge set* E, along with a function f from E to $\{\{u, v\} \mid u, v \in V, u \neq v\}$. Edges e_1 and e_2 are *multiple edges* if $f(e_1) = f(e_2)$.

Definition 7.1.3. A *pseudograph* $G = (V, E)$ consists of a vertex set V, edge set E, and a function f from E to $\{\{u, v\} \mid u, v \in V\}$. An edge is a *loop* if $f(e) = \{u, u\}$ for some $u \in V$.

Definition 7.1.4. A *directed graph* (V, A) consists of a vertex set V and s set A of ordered pairs of elements of V called *arcs*.

Definition 7.1.5. A *directed multigraph* $G = (V, A)$ consists of a vertex set V, and arc set A, and a function f from A to $\{\{u, v\} \mid u, v \in V\}$. Arcs e_1 and e_2 are multiple arcs if $f(e_1) = f(e_2)$.

Example. Page 444 Number 20.

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