Section 7.2. Graph Terminology

Note. In this section we increase our vocabulary concerning graphs.

Definition 7.2.1. Two vertices u and v in an undirected graph G are called *adjacent* (or *neighbors*) in G if $\{u, v\}$ is an edge of G. If $e = \{u, v\}$, then edge e is *incident* with vertices u and v. Edge e is said to *connect* u and v. Vertices u and v are *endpoints* of edge $\{u, v\}$.

Definition 7.2.2. The *degree* of a vertex, denoted $\deg(v)$, in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex.

Theorem 7.2.1. The Hand Shaking Theorem.
Let $G = (V, E)$ be an undirected graph with e edges. Then $2e = \sum_{v \in V} \deg(v)$.

Example. Page 454 Number 6.

Theorem 7.2.2. An undirected graph has an even number of vertices of odd degree.

Definition 7.2.3. When (u, v) is an arc of digraph G, u is *adjacent to* v and v is *adjacent from* u. u is the *initial vertex* and v is the *terminal vertex* of arc (u, v).

Definition 7.2.4. In a digraph, the *in-degree* of a vertex v, denoted $\deg^-(v)$, is the number of arcs with v as their terminal vertex. The *out-degree* of v, denoted $\deg^+(v)$, is the number of edges with v as their initial vertex.

Example. Page 454 Number 8.

Theorem 7.2.3. Let $G(V, A)$ be a digraph. Then

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |A|.$$

Definition 7.2.5. A simple graph G is *bipartite* if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2.

Example. Page 455 Number 14.

Definition 7.2.6. A *subgraph* of $G = (V, E)$ is a graph $H = (W, F)$ wher e$W \subseteq V$ and $F \subseteq E$.

Definition 7.2.7. The *union* of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$ and is denoted $G_1 \cup G_2$.

Example. Page 455 Number 30.

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