Section 7.3. Representing Graphs and Graph Isomorphisms

Note. In this section we consider some properties of undirected graphs.

Definition. Suppose $G = (V, E)$ is a simple graph where $|V| = n$ and $V = \{v_1, v_2, \ldots, v_n\}$. The *adjacency matrix* A of G is the $n \times n$ 01 matrix with 1 as its (i, j)th entry when v_1 and v_j are adjacent and 0 otherwise.

Examples. Page 464 Numbers 6 and 12.

Note. The adjacency matrix of an (undirected) graph is symmetric.

Note. We can analogously define the adjacency matrix for a digraph:

$$\alpha_{ij} = \begin{cases} 1 \text{ if } (v_i, v_j) \in A \\ 0 \text{ otherwise.} \end{cases}$$

Example. Page 464 Number 20.

Definition. Let $G = (V, E)$ be an undirected graph. Suppose $v_1, v_2, \ldots, v_n$ are the vertices and $e_1, e_2, \ldots, e_m$ are the edges of G. The *incidence matrix* $M = [m_{ij}]$ is

$$m_{ij} = \begin{cases} 1 \text{ when } e_j \text{ is incident with } v_i \\ 0 \text{ otherwise.} \end{cases}$$

Example. Page 464 Number 26.

Definition 7.3.1. The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there is a one to one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2. f is called an *isomorphism.*

Examples. Page 465 Numbers 34 and 42.

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