Section 7.4. Connectivity

Note. In this section we define several ideas related to the concept of connectivity of a graph based on paths.

Definition 7.4.1. A *path* of length n from vertex u to vertex v ($n \in \mathbb{N}$) in an undirected graph is a sequence of edges $e_1, e_2, \ldots, e_n$ of the graph such that $f(e_1) = \{x_0, x_1\}$, $f(e_2) = \{x_1, x_2\}, \ldots, f(e_n) = \{x_{n-1}, x_n\}$ where $x_0 = u$ and $x_n = v$. We may denote the path by the vertex sequence $x_0, x_1, \ldots, x_n$. The path is a *circuit* if $u = v$.

Example. Page 473 Number 8.

Note. A *path* in a digraph is similarly defined (see Definition 7.4.2 in the text).

Definition 7.4.3. An undirected graph is *connected* if there is a path between every pair of distinct vertices of the graph.

Example. Page 475 Number 31.

Note. We can set up an equivalence relation on the vertices of a graph by saying aRb is either $a = b$ or there is a path from a to b. The equivalence classes of this relation are called *connected components* of the graph.

Example. Page 475 Number 26.

Definition. If the removal of a vertex and all edges incident with it from a connected graph forces a disconnected graph, then the vertex is called a *cut vertex*.

Definition. If the removal of an edge from a connected graph produces an unconnected graph, the edge is a *cut edge* or *bridge*.

Definition 7.4.4/7.4.5. A digraph is *strongly connected* if there is a path from a to b for all distinct vertices a and b. A digraph is *weakly connected* if there is a path between any two distinct vertices in the underlying undirected graph.

Theorem 7.4.2. Let G be a graph or digraph with adjacency matrix A (with respect to vertex ordering $v_1, v_2, \ldots, v_n$). The number of different paths of length r from v_i to v_j, where r is a positive integer, is the (i, j)th entry of A^r.

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